BEAM STABILITY ANALYSIS FOR ADS

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ABSTRACT
In an Accelerator Driven System (ADS) aimed for the transmutation of nuclear waste, supercriticality accidents are eliminated but are replaced instead by over-power transients. Thus an important challenge for designing an ADS-Reactor facility is to demonstrate that the multiplicity of the target cannot change significantly due to some perturbations. This requires understanding the underlying physics of the spallation process, but also of the collective effects such as the space charge effects that tend to modify the final beam distribution on the target. As part of these investigations, space charge simulations of the KURRI scaling FFAG accelerator in Japan have been undertaken, using a newly implemented module in the tracking code Zgoubi, that allows the computation of self fields. A perturbative approach is used to investigate the beam stability. This is followed by MCNP6 simulations to determine the variation of the spallation induced neutrons that will drive the sub-critical reactor assembly.

KEYWORDS
ADS, overpower, space charge, stability

1 INTRODUCTION
It has been shown that an ADS system aimed to burn the nuclear waste has a transmutation performance per unit of energy produced about 20 times higher than fast critical reactors [1]. The main advantage of an ADS being the external source of neutrons that drives the reactor core. However, the R&D effort required to bring these systems into maturity is still important. Among the issues is the licensing of the high power accelerator with the aim of producing nuclear energy. There are no clear guidelines of what the requirements must be with respect to this. This requires a careful assessment of the potential accidents.

2 BEAM POWER REQUIREMENTS AND IMPORTANCE OF THE SOURCE NEUTRONS
Although the external source of neutrons does not have an impact on the $k_{eff}$ of the subcritical blanket, it is important to realize that the multiplication of the spallation neutrons may be so different from that of the neutrons that are born in the fuel elements from fission or other processes.
Therefore, it is important to make this distinction between $k_{eff}$ and $k_s$ where $k_s$ represents the source multiplication factor. The main question that raises is whether the safety of an ADS may be affected by the way the first generation neutrons multiply in the core.

### 2.1 Beam power requirements

If $N_0$ is the number of primary neutrons injected in the core per proton, from the spallation source ($N_0$ is an average quantity), and $k_i$ is the multiplication factor of the generation $i$, then the number of prompt fission neutrons after multiplication is:

$$N = N_0 (1 + k_1 + k_1 k_2 + k_1 k_2 k_3 + \ldots + k_1 k_2 \ldots k_p + \ldots) - N_0$$

$$= N_0 k_1 (1 + k_2 + k_2 k_3 + k_2 k_3 k_4 + \ldots + k_2 k_3 \ldots k_p + \ldots)$$

The first neutron generations have a multiplication factor $k_i$ that differs from $k_{eff}$, the multiplication factor of the blanket: such a distinction is important since the spectrum of the fission neutrons is highly dependent on the spectrum of the neutrons from which they originated.

Each fission produces $\nu$ neutrons ($\sim 2.5$). Thus, the total number of fissions per proton is:

$$N_f = \frac{N}{\nu} = \frac{N_0}{\nu} k_1 S$$

where

$$S = 1 + k_2 + k_2 k_3 + k_2 k_3 k_4 + \ldots + k_2 k_3 \ldots k_p + \ldots$$

The thermal power of the core is thus given by:

$$P_{th,c} = E_f (MeV) I (A) \frac{N_0}{\nu} k_1 S + P_{dh}$$

where $E_f$ is the fission energy, $I$ is the beam current and $P_{dh}$ is the power of the decay heat: the decay of the short-lived radioisotopes created in fission such as the fission products and the actinides continues even if the accelerator is shut-down, i.e (I=0). This can represent up to 7% of the blanket thermal power, immediately after shutdown. For simplicity, we will assume here that $P_{dh} \sim 0$.

Another assumption in this analysis is that all the spallation neutrons injected in the core per second multiply in the same way. In other words, all series (defined by Eq. (3)) which originate from a primary proton converge in the same way: this is generally not the case and is mainly dependent on the time structure of the proton beam. Besides, no reactivity feedback effects are considered here. A more detailed analysis of this will be provided in a later paper.

### 2.2 Importance of the source neutrons

A major advantage of ADS is its enhanced safety: if the sub-criticality level (or $k_{eff}$) is far below 1 (this is generally chosen in order to accommodate for any possible positive reactivity insertion), super-criticality accidents are eliminated. However, it follows from Eq. (1) that over-power transients can be a serious issue: if the proton beam power ($N_0 I$) and/or the neutron source amplification ($k_1 S$) change, this implies that the thermal power of the core will follow as well. Therefore, it is crucial to understand the effect of the first generations of neutrons on the reactor power. Fig 1 illustrates the importance of the first neutron generation $k_1$ which, for a given power, can lower the $k_{eff}$ and thus increase the margin from criticality in an ADS. To have a better understanding of the source amplification, we will calculate the amplification term $S$ defined in Eq. (3) above using...
Figure 1: Core thermal power as a function of $k_{eff}$ and $k_1$. Here we assume that $S = \frac{1}{1 - k_{eff}}$ so that $P_{th} \propto \frac{k_1}{1 - k_{eff}}$

a simplified model\[1].

Without any loss of generality, we define the sequence $b_i$ as follows:

$$
k_{i+1} = (1 + b_i)k_{eff}, \quad 1 \leq i \leq p - 1
$$

$$
k_i = k_{eff}, \quad i > p
$$

The sequence $b_i$ takes into account the importance of the first generation neutrons which have a multiplication factor different from $k_{eff}$ up to the generation number $p$. $p$ here can be made as big as possible. So, for the moment, the only assumption made is that it takes $p$ generations before $k_i$ converges to $k_{eff}$.

It follows,

$$
S = 1 + k_2 + k_2k_3 + k_2k_3k_4 + ... + k_2k_3k_p + ...
$$

$$
= 1 + \left[ (1 + b_1)k_{eff} + (1 + b_1)(1 + b_2)k_{eff}^2 + ... + (1 + b_1)(1 + b_2)\cdots(1 + b_{p-1})k_{eff}^{p-1} \right] \\
+ \left[ (1 + b_1)(1 + b_2)\cdots(1 + b_{p-1})k_{eff}^p + (1 + b_1)(1 + b_2)\cdots(1 + b_{p-1})k_{eff}^{p+1} + ... \right]
$$

$$
= 1 + \sum_{i=1}^{p-1} \left[ \prod_{j=1}^{i} (1 + b_j) \right] k_{eff}^i + \left[ \prod_{j=1}^{p-1} (1 + b_j) \right] \frac{k_{eff}^p}{1 - k_{eff}}
$$

(6)

Then, by adding and subtracting, $S_0 = \sum_{i=0}^{\infty} k_{eff}^i = \frac{1}{1 - k_{eff}} = 1 + \sum_{i=1}^{p-1} k_{eff}^i + \frac{k_{eff}^p}{1 - k_{eff}}$, one obtains:

$$
S = \frac{1}{1 - k_{eff}} + \sum_{i=1}^{p-1} \left[ \prod_{j=1}^{i} (1 + b_j) - 1 \right] k_{eff}^i + \left[ \prod_{j=1}^{p-1} (1 + b_j) - 1 \right] \frac{k_{eff}^p}{1 - k_{eff}}
$$

$$
= S_0 + \delta S
$$

(7)

\[1\] A rigorous approach to investigate the kinetics of subcritical multiplying systems relies on the adjoint flux calculation \[2\], where one uses a weighting function to evaluate the “importance” of producing fission neutrons.
The perturbation introduced is contained in the term $\delta S$: the first term of the latter represents the impact of the perturbation up to the generation number $p$. However, the last term accounts for the additional impact of the perturbation that is carried out by the later generations that converged to $k_{eff}$. Since the fission process is a chain reaction, the memory of the system is somehow conserved. Now, the thermal power of the core rewrites:

$$P_{th.c} = E_f(MeV)I(A)\frac{N_0}{\nu}k_1S_0 + E_f(MeV)I(A)\frac{N_0}{\nu}k_1\delta S + P_{dh}$$

where $P_0$ represents the contribution of the neutrons which undergo fission reactions with a multiplication factor $k_i = k_{eff}$ while $\delta P$ represents the excess or deficiency of the neutrons which undergo fission reactions with a multiplication factor $k_i \neq k_{eff}$.

One should point out here that $k_{eff}$ is an average quantity. In a critical reactor where there is no external source of neutrons, $k_{eff}$ should remain close to 1 although the neutrons multiply in different ways depending on their spectrum and location in the core. In an ADS where the source is the driving term, the fact that the neutrons multiply in different ways makes the analysis far more complicated and intricate: the history of the spallation neutrons in the core which are the source of all forthcoming neutrons has an impact such that, if the multiplication factor $k_1$ of the first generation neutrons multiplies by 2, the thermal power of the core will double as well. This shows the importance of tailoring the spectrum of the target to maximize its efficiency. Although the spallation neutrons represent a small fraction of the total neutrons in the core, it allows more degrees of freedom to play with, that cannot be otherwise allowed in a critical reactor, since it would jeopardize its safety.

Now, let us assume that $b_j = \frac{1}{j}$ and inject it into Eq. (7). This form of $b_j$ was chosen in order to fit an example of the evolution of the multiplication factor as a function of the generation number (Fig. 1 in [3]). It follows:

$$S = \frac{1}{1 - k_{eff}} + \sum_{i=1}^{p-1} i.k_{eff}^i + (p - 1) \frac{k_{eff}^p}{1 - k_{eff}} ; \quad p \geq 2 \quad \text{(8)}$$

Fig. 2 illustrates the impact of the perturbation of the first neutron generations on the core power change as a function of the sub-criticality level: one can observe that, for deeply sub-critical systems, the effect is less severe. In addition, this study revealed that the most important effect of the perturbation is carried out by the later neutron generations that converged to $k_{eff}$ (solid vs dashed lines). In order to understand this result, the idea is to study the convergence rate of the series $S_0 = \sum_{i=0}^{\infty} k_{eff}^i = 1/(1 - k_{eff})$.

### 2.3 Convergence rate and the concept of neutron multiplication time

The convergence rate of the order $p$ is defined as the relative error of the sum $S_0$ by estimating only the first $(p-1)$ terms of this series:

$$\epsilon = \frac{S_p}{S_0} = \left( \frac{1}{1 - k_{eff}} - \frac{1 - k_{eff}^p}{1 - k_{eff}} \right) \times (1 - k_{eff})$$

$$= k_{eff}^p \quad \text{(9)}$$
Figure 2: Power variation as a function of $k_{\text{eff}}$ by perturbing the first $p$ terms of the series: $p=3$ to $p=9$ are shown here: $\frac{\delta P_{th}}{P_{th0}} = \frac{\delta S}{S_0}$ is shown by the solid lines. The dashed lines show the effect of the perturbation limited to the generation number $p$.

Figure 3: Convergence of the amplification factor $S_0$ as a function of the neutron generation number.

Fig. 3 shows the convergence rate of the series for different multiplication factors. For instance, if $k_{\text{eff}} = 0.95$, the error is down to 0.6% after 100 neutron generations. As the multiplication factor $k_{\text{eff}}$ becomes higher and higher ($k_{\text{eff}} \sim 1$), more time is needed by the system to converge to the final state level where $S_0 = 1/(1 - k_{\text{eff}})$. Therefore the fission process continues for a long time after the injection of the spallation neutrons because the contribution of forthcoming generations becomes large. Thus, perturbing the first neutron generations will have an important impact mainly for slowly-converging systems, i.e. for nearly critical systems as shown in Fig. 2.

Based on the results above, one defines the multiplication time $T_m$ as the time it takes for the spallation neutrons injected into the core to multiply and reach the amplification factor $S_0$ within a specified accuracy. It is a measure of the duration of the lasting fission process induced by the injection of the spallation neutrons into the core. Then, according to Eq. (9), it results

$$T_m = \frac{\ln(\epsilon)}{\ln(k_{\text{eff}})} T_g \propto \frac{1}{|\ln(k_{\text{eff}})|} \quad ; \quad k_{\text{eff}} \lesssim 1$$

(10)

where $T_g$ is the mean generation time, i.e. the time between two consecutive neutron generations in the core. This result illustrates the importance of $k_{\text{eff}}$ on the safety of an ADS: if $k_{\text{eff}}$ is very close to 1, the reliability of the accelerator is no longer problematic since the fission process continues...
for a long time after the beam is off. However, this poses a serious safety issue and cannot be tolerated. One remedy to this problem is to use a multi-beam target system: this improves the transmutation characteristics which become more uniform, but also reduces the risk induced by uncontrolled perturbation or damage of one of the external neutron sources.

2.4 Interplay between the different parameters and feedback effects

In general, it is assumed that the power control of an ADS can be achieved through the control of the beam current of its accelerator. This is expected from Eq. (4).

However, space charge effects may become of major concern when increasing the beam current: collective effects, which often manifest through an emittance increase and beam halo, tend to modify the final beam size and distribution on the target. Therefore, the question that raises is whether there may be an interplay between the target multiplicity \( N_0 \) and the beam current in Eq. (4).

One particularly interesting situation to happen would be the case where an unexpected increase of the beam intensity on the target induces a self-regulating effect that manifests through a decrease of the multiplicity. The idea would be to exploit the collective effects, such as the space charge effects for this purpose: an inadvertent increase of the beam current and/or its energy would modify the final beam shape and distribution on the target through the space charge forces. If this effect is non negligible and can be accounted for in an easy way, then the design of the transport line between the accelerator and the target should be such that this effect is negative, i.e an increase of the beam current induces, to certain extent, a decrease of the target multiplicity. The importance of this, from the viewpoint of safety, is crucial, since then one can demonstrate that the accelerator can provide a feedback effect, similar to the Doppler effect in the reactor theory.

3 STABILITY ANALYSIS FOR FFAG

The KV space charge kick module [4] implemented in the tracking code ZGOUBI [5] is used to carry out a parametric study of the space charge effects in an FFAG, similar to the KURRI 150 MeV scaling FFAG [6]: the magnetic field is perturbed in order to scan the un-depressed phase advance in both planes and compute the change in the number of betatron oscillations, what is generally referred to as the tune shift. The latter is averaged over the entire lattice: a KV beam distribution is generated on several closed orbits between injection and extraction, from which the average value of the betatron oscillations of the cell is computed. The shift in the frequency of betatron oscillations is thus deduced. In order to interpret the results, one recalls the Laslett tune shift formula:

\[
\Delta Q_y = \frac{1}{4\pi} \int_0^C \beta_y(s) \frac{2Q}{r_y(r_x + r_y)} ds 
\]

Interchanging x and y gives the horizontal tune shift. After simplification, this yields:

\[
\frac{\Delta Q_y}{Q_y} \propto \frac{r_y^3}{r_x + r_y} \\
\frac{\Delta Q_x}{Q_x} \propto \frac{r_x^3}{r_x + r_y}
\]

It results that:

\[
Q_x \nearrow \text{ or } Q_y \searrow \implies \frac{\Delta Q_x}{Q_x} \searrow
\]

AccApp ’15, Washington, DC, November 10-13, 2015
and vice versa. As can be seen in Fig. 4, beam tracking results are in good agreement with the above scaling laws in both planes. Although an important emittance increase is observed in the region where \( Q_x = 0.33 \), due to the proximity of the 3rd integer resonance, this requires further investigation.

![Figure 4: Contour plot of the space charge induced tune shift as a function of the undepressed phase advances per cell, \( Q_x/N \) and \( Q_y/N \) where \( N \) is the number of sectors.](image)

### 3.1 Beam distribution on the target

In order to look at the effect of several beam distributions and sizes on the target multiplicity, Monte Carlo simulations are carried out, using MCNP6 [8]. A cylindrical lead target of 30cm length is simulated on which a 1 GeV proton beam impinges. One first parameter to vary is the beam size. In this case, the beam has a uniform distribution with a square cross section. As shown in Fig. 5, increasing the beam size decreases the neutron production of the target. This effect is non-negligible and can reach a few percent, depending on the ratio of the beam size to target size. One can explain this result via examination of the interaction probability of a proton with the target material: the probability that a non-elastic nuclear collision occurs in the target is calculated using the following equation:

\[
P_n = 1 - \exp\left(\frac{-R}{\lambda}\right) \tag{14}
\]

where \( R \) is the range of the proton (\( g/cm^2 \)) and \( \lambda \) is the non-elastic nuclear collision mean free path. In order to maximize this probability, there are essentially two parameters to play with: the proton range, which is fixed by the proton energy for a given material, as well as the probability of the collision itself to occur which is determined by \( \lambda \). The latter depends also on the energy of the incident particle and is almost constant above 100 MeV [7]. Another hidden parameter is the available target material for each particle to travel across and produce inelastic collisions. In other words, if the proton range exceeds the size of the target, then the spallation process becomes inefficient. Furthermore, if the target size largely exceeds the proton range, then it is likely that a non-negligible fraction of the spallation produced neutrons be absorbed by the target material before escaping towards the reactor core. From that point of view, the target has to be adequately sized.
to maximize the neutron production and a bunch of particles spread over different locations with respect to their center of mass will exhibit a behavior such as the outer particles generally have a lower probability to interact than the centroid.

In order to investigate the effect of the beam distribution on the neutron multiplicity, one used the concept of RMS equivalent beams that was introduced by Sacherer and Lapostolle in 1971 [9]: according to this concept, two beams are equivalent in an approximate sense if the second moments of the distribution are the same. The results from Monte Carlo simulations seem to indicate that using a uniform or gaussian distribution does not have an impact on the target multiplicity. However, the neutron flux distribution indicates some differences. This requires further investigation.

4 CONCLUSION

In this paper, one presented a simplified model to investigate the impact of the perturbation of the source neutrons on the thermal power of the core. This showed the importance of the source neutrons which allow more degrees of freedom that cannot be tolerated in critical reactors. Two key parameters that come to play in an ADS are the accelerator beam power and the target multiplicity. The stability analysis focused on the question of the interplay between these two parameters, by means of a change in the beam size and/or its distribution via space charge forces. This effect can be important, of the order of few percent and, if well mastered, can demonstrate the reactor stability \textit{vis-à-vis} overpower accidents.

References


